AN INTEGRAL EQUATION APPROACH FOR ANALYSIS OF CONTROL CHARTS

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I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Yupaporn Areepong

Signature of Candidate

Dedication

To Dad & Mom

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Abstract

This thesis is concerned with the use of Statistical Process Control (SPC) charts for detection of change-point in distributions in quality control and surveillance problems. We derive explicit analytical formulas and develop numerical algorithms for evaluating important characteristics of "Exponentially Weighted Moving Average" (EWMA) control charts for a range of distributions.

The most popular characteristics of a control chart are Average Run Length (ARL) - the mean of observations/times that are taken before a system is signalled to be out-of-control when it is actually still in-control, and Average Delay (AD) time - the mean of delay of true alarm times before a system that is actually out-of-control is signalled to be out-of-control. An important property required of ARL is that it should be sufficiently large when the process is in-control to reduce a number of false alarms. On the other side, if the process is actually out-of-control then its AD should be as small as possible. Traditional methods that are used for evaluating chart characteristics include Markov Chain Approach (MCA), Integral Equation (IE) and Monte Carlo simulation (MC) methods. Some crucial features of the methods are as follows: the MCA requires many matrix inversions and there is no theoretical proof of convergence of the method; the IE is most advanced method and it was used before only for Gaussian distribution; the MC is very time consuming.

In this thesis, we find explicit formulas for ARL and AD of EWMA in the case when observations are exponentially distributed. These explicit formulas can be applied to some other distributions, e.g. the Pareto distribution. The numerical results obtained from our explicit formulas are compared with results obtained from the Monte Carlo simulation (MC) and Markov Chain Approach (MCA). We also compare the performance of the EWMA procedure with charts obtained with the CUSUM and Shiryayev-Roberts procedures. The technique that we use to derive the formulas for an exponential distribution cannot be used to derive formulas for gamma and Weibull distributions. However, we have developed a different method for evaluating the ARL and AD for the case of gamma and

Weibull distributions. This method is based on a numerical solution of Integral Equations based on Gauss-Legendre integration rules to approximate the integrals. Numerical results for these distributions are compared with results from other approaches.